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Theoretical note

On the minimal relative motion principle—lateral displacement of a contracting bar

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Abstract

Xausa, Beghi, and Zanforlin (J. Math. Psychol. 45(4) (2001) 635) provide an account of perceptual organization based on their 'minimal relative motion' principle. They claim that this principle can account for the percept generated by a contracting bar that is simultaneously translating laterally. We critique the mathematical analysis provided in the aforementioned paper. We conclude that the 'minimal relative motion' principle, in the form presented, cannot adequately explain the percept reported by observers.

Keywords: Stereokinetic effect; Structure from motion; Perceptual organization; Rigidity assumption; Motion minimization

1. Introduction

Xausa et al. (2001) report that a bar that is contracting over time as it moves laterally appears to be both rotating and translating into the frontal plane. This is an interesting phenomenon. Although in theory a number of percepts are compatible with this stimulus, the human visual system consistently selects one interpretation.

To account for this percept, Xausa et al. (2001) postulate a 'minimal relative motion constraint'. This constraint as defined by Xausa et al. (2001) forces all points to achieve a common speed that is set equal to or larger than the length of the maximum velocity vector present in the physical stimulus.

Under orthographic projection, points on the translating and contracting bar can achieve this common motion by assuming an invisible motion component of each point into the frontal plane. In reference frame $Oxyz$, the motion of the midpoint (M) in the frontal plane is always smallest compared to all other points on the stimulus; such a relationship between M and all other points implies that M would exhibit a larger motion into the frontal plane than any other point. There can be no combination of rigid rotation and translation that would be compatible with this type of motion of the bar.

2. Mathematical model

The stimulus that Xausa et al. (2001) describe is a bar that contracts in the Oz direction as it translates in the Ox direction, where Oxz is the frontal plane (Fig. 1).

Xausa et al. (2001) describe different 'profiles' of contraction and translation, but our discussion will cover all of these, without loss of generality.

This stimulus results in the percept of a bar translating in depth (in the Oy direction) as it is rotating in a plane perpendicular to the Ox-axis.

Xausa et al. (2001) state that the "minimal relative motion principle minimize(s) the differences between the length of the velocity vectors of [...] points" (p. 640). More specifically, they state that they add a motion component in depth (along the Oy-axis) to every point such that "the length of the resulting velocity vector is constant in time and the same for all points" (p. 640). Thus they assign motion in depth such that all points move with equal speed, but not necessarily equal direction.

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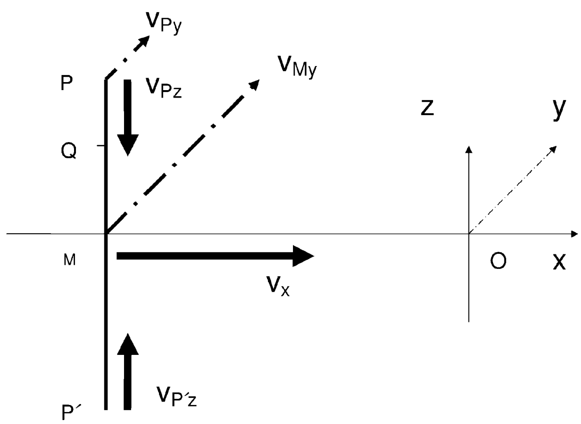

Fig. 1. Sketch of a bar PP' moving laterally along the Ox-axis as it is contracting around midpoint M. Dashed lines represent the (projectively invisible) depth component. Q is an arbitrary point on the bar. Based on the 'minimal relative motion' principle, velocity components in depth closer to the Ox-axis (v_{My}) have greater length than those farther away (v_{Py}).

2.1. A relative coordinate system

In reviewer comments, the authors of Xausa et al. (2001) provide an intuition on how they derive the 'minimum relative motion constraint'. Xausa et al. (2001) define a coordinate frame relative to P (Fig. 2a). In this coordinate frame, P has the smallest velocity (i.e. zero) whereas P' has the largest velocity in the image plane. Motion in depth is then assigned such that the lengths of the resulting velocity vectors in this coordinate frame are equal for all points. As a result, motion in depth should be largest for P and smallest for P' (Fig. 2b). Finally, the common (translation) component of motion in depth is subtracted out, which appears to leave a rotation of the bar in depth (Fig. 2c).

To minimize relative motion, Xausa et al. (2001) define a "velocity length" (p. 640) that points on the stimulus should possess

$$v_{\max} = \sqrt{v_{x_{\max}}^2 + 4v_{z_{\max}}^2}, \tag{1}$$

where $v_{x_{\max}}$ and $v_{z_{\max}}$ are, respectively, the maximum velocity in the Ox and Oz direction over time in the $Oxyz$ coordinate frame. $v_{\max}$ takes the above form since the motion of P' relative to P in the relative coordinate system equals $2v_{z_{\max}}$. The velocity component in depth is then assigned to points on the bar in such a way that the motion in the relative coordinate frame equals $v_{\max}$ for all points.

$$v_{Qx}^2(t) + v_{Qy}^2(t) + v_{Qz}^2(t) = v_{\max}^2, \tag{2}$$

where all values are known based on the physical stimulus with the exception of $v_{Qy}(t)$.

After we obtain the velocity in depth $v_{Qy}(t)$ from Eq. (2) it is split up in a translational component and a rotational component. The translational component is set equal to the motion of the midpoint of the bar v_{My}. The residual motion is the rotational component, and is thus defined relative to v_{My}.

To express the speed of an arbitrary point Q on the bar in relationship to its midpoint M and endpoint P, Xausa et al. (2001) define

$$\frac{MQ}{MP} = \lambda, \text{ where } -1 \leqslant \lambda \leqslant 1 \tag{3}$$

from which Xausa et al. (2001) claim

$$v_{Qz}(t) = \lambda v_{Pz}(t). \tag{4}$$

However, when introducing these equations, Xausa et al. (2001) have changed from the relative coordinate system to the absolute ($Oxyz$) coordinate system, passing through a coordinate system that is relative to $(v_{Mx}, v_{My}, 0)$.

Vector lengths are not conserved under these coordinate system changes. That is, subtracting out common translations for all points does not conserve the ratio of vector lengths between different points on PP', contrary to the intuition provided in Fig. 2. We plot the speeds under these different reference frames in Fig. 3 assuming $v_{Px} = v_{Pz}$ in $Oxyz$. We also note that the intuition provided in Fig. 2, where speed is constant in the relative coordinate system, is different from Eq. (2), where speed is constant in the absolute coordinate system.

Fig. 3 shows that a constant common speed for all points on the bar is maintained only in a reference frame that moves relative to the absolute reference frame with a velocity of $(v_{x_{\max}}, 0, -v_{Pz})$ or $(v_{x_{\max}}, 0, v_{Pz})$. There is no common constant speed in the other two reference frames. It may appear that if not constant in $Oxyz$, v is at least almost linear with λ, allowing an interpretation of this motion as a form of translation plus rotation. However, this is due to the fact that $|v|$ only conveys information about speed, not direction. Since all three curves in Fig. 3 describe the same motion, the predicted non-rigidity of the motion is most apparent in the nonlinearity of the curve relative to $(v_{Mx}, v_{My}, 0)$. We will illustrate the issue of non-rigidity from a different perspective next.

2.2. The absolute coordinate system

It is our opinion that the problem with the argument put forth by Xausa et al. (2001) arises when Xausa et al. (2001) attempt to equalize the speed of all points on the bar to some $v_{\max}$. It follows from Eqs. (2) and (4) for any definition of $v_{\max}$ that

$$|v_{y,\lambda}(t)| = \sqrt{v_{\max}^2 - v_{Px}^2(t) - \lambda^2 v_{Pz}^2(t)}, \tag{5}$$

where $v_{Px}(t)$ equals $v_{x,\lambda}(t)$ for any λ.

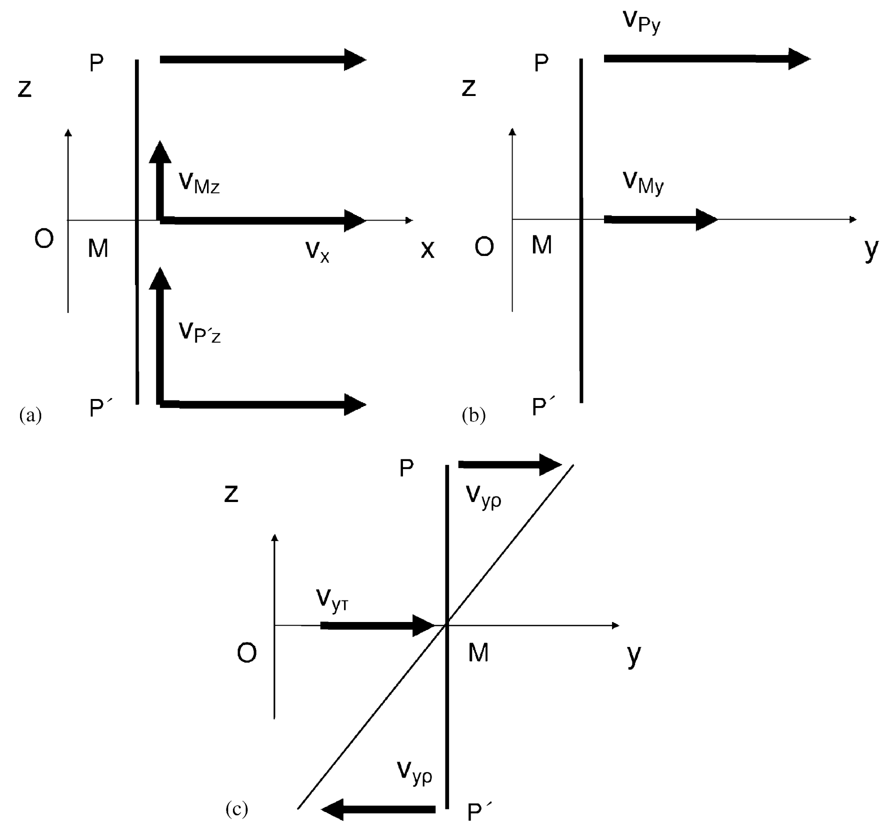

Fig. 2. Sketch of the 'minimum relative motion principle' as outlined by Xausa et al. (2001). Motion is minimized in a coordinate frame relative to P (a). Under the 'minimum relative motion constraint', perceived motion in depth should decrease as we move from P to P' (b). Motion in depth is then separated into a translational ($v_{y\tau} = v_{My}$) and a rotational (v_{y_ρ}) component relative to v_{My} (c). Adapted from reviewer comments by M. Zanforlin.

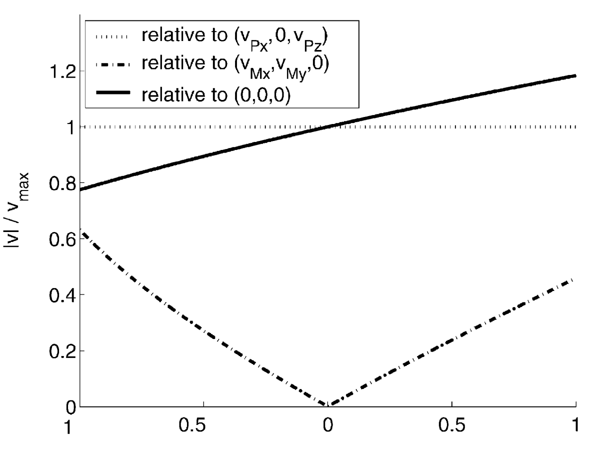

Fig. 3. Speed of points relative to $v_{\max}$ as a function of position (λ) in three different reference frames. We assume $v_{Px} = v_{Pz}$ in $Oxyz$. In a reference frame whose velocity relative to the absolute reference frame is $(v_{Px}, 0, -v_{Pz})$, all points have a common speed $v_{\max}$. However, this is not the case in either the absolute reference frame or a reference frame that moves with the midpoint M of the stimulus. We first obtain $|v|$ in the absolute reference frame $Oxyz$. We can then obtain $|v|$ in the relative reference frame used by Xausa et al. by subtracting $(v_{Px}, 0, -v_{Pz})$ from all points or in a reference frame relative to the midpoint M of the stimulus by subtracting $(v_{Mx}, v_{My}, 0)$ from all points.

It is easy to see that $|v_{y,\lambda}(t)|$ is maximal when $\lambda = 0$. Thus $|v_{My}(t)| > |v_{y,\lambda \neq 0}(t)|$. However, there is no combination of translation and rotation of a rigid object where the midpoint M of an object has greater speed in depth than any other point on the object. In that case the stimulus should appear to 'break' at its midpoint. Therefore, the 'minimal relative motion' principle, as presented, cannot adequately explain the percept of simultaneous translation and rotation of a rigid object reported by observers.

Xausa et al. (2001) try to rescue the rigidity of the bar by using $\mathrm{sign}(\lambda)$ to determine the sign of the rotational component ρ of motion in depth

$$v_{y_\rho,\lambda}(t) = \mathrm{sign}(\lambda)|v_{y,\lambda}(t) - v_{My}(t)|. \tag{6}$$

Xausa et al. (2001) choose this formulation based on the direction of motion human observers report for the top and bottom parts of the bar, rather than on any mathematical considerations. However, given that the translational component of motion in depth is equal for all points, this implies that $|v_{My}(t)|$ is smaller than $|v_{y,\lambda \neq 0}(t)|$ for half of the points on the stimulus. This directly contradicts Eq. (5).

3. Conclusion

Based on the mathematical application of the 'minimal relative motion' principle as set forth in Xausa et al. (2001), a percept should be achieved that does not correspond to what is reported in the experimental section of that same paper. It is reported that subjects perceive a mixture of translation in depth and rotation in a plane perpendicular to the horizontal axis on presentation of a contracting bar that is laterally translating. In contrast, based on the mathematical derivation supplied by Xausa et al. (2001), subjects should not report the percept of a rotating bar receding in depth.

Though the term 'minimal relative motion' may suggest that motion is being minimized, not equalized, Xausa et al. (2001) explicitly state that the 'minimal relative motion' principle assigns motion in depth such that "the length of the resulting velocity vector is constant in time and the same for all points" (p. 640).

The non-rigidity of the predicted percept is not a direct result of the particular 'common motion' $v_{\max}$ that Xausa et al. (2001) have chosen. Choosing a different 'common motion' will not remediate the problem. It is impossible for a rotating bar to have the same speed everywhere while simultaneously remaining rigid.

Any account which equalizes the speed of all points will run into the same problem. Such accounts have to assume that the perceived motion into the frontal plane of the midpoint M of an object is larger than the perceived motion of any other point on the object.

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